Supersingular Isogeny Key Encapsulation

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SIKE

Supersingular **I**sogeny **K**ey **E**ncapsulation (SIKE)

- IND-CCA2 KEM
- Based on **S**upersingular **I**sogeny **D**iffie-**H**ellman (SIDH)
- Uses Hofheinz et al. transformation (TCC 2017) on SIDH to achieve CCA security

The SIKE protocol specifies:

- Parameter sets
- Key/ciphertext formats
- Encapsulation/decapsulation mechanisms
- Choice of symmetric primitives (hash functions, etc.)

A brief history of SIDH

Couveignes, *Hard Homogeneous Spaces* (1996), ePrint:2006/291

- First explicit mention of isogenies in cryptography
- Unpublished until 2006

Galbraith, *Constructing isogenies between elliptic curves over finite fields* (1999)

- First published cryptanalysis of isogeny problem

Jao and Venkatesan, *Use of isogenies for design of cryptosystems* (2003), US 7499544 (assignee: Microsoft Corporation)

- First (only?) patent on isogeny-based cryptography
- Does not apply to SIDH
- SIDH/SIKE is, to our knowledge, patent-free

Charles et al., *Cryptographic hash functions from expander graphs* (2009)

- First use of supersingular isogenies in cryptography

A brief history of SIDH

Stolbunov, *Constructing public-key cryptographic schemes based on class group action on a set of isogenous elliptic curves* (2010)

- First published isogeny-based public-key cryptosystem
- Essentially identical to Couveignes' unpublished 1996 work
- Partially broken by Childs, Jao, and Soukharev (2014)

Jao and De Feo, *Towards quantum-resistant cryptosystems from supersingular elliptic curve isogenies* (2011)

- Invention of SIDH
- First supersingular isogeny-based public-key cryptosystem

Galbraith et al., *On the Security of Supersingular Isogeny Cryptosystems* (2016)

- Active attack against SIDH with static key re-use
- Necessitates use of Hofheinz et al. transform for CCA security

Overview of SIDH

1. Public parameters: Supersingular elliptic curve E over F.
2. Alice chooses a kernel $A \subset E$ and sends E/A to Bob.
3. Bob chooses a kernel $B \subset E$ and sends E/B to Alice.
4. The shared secret is

$$E/\langle A, B\rangle = (E/A)/\phi_A(B) = (E/B)/\phi_B(A).$$

$$\begin{tikzcd}
E \arrow[r, "\phi_A"] \arrow[d, "\phi_B"'] & E/A \arrow[d] \\
E/B \arrow[r] & E/\langle A, B\rangle
\end{tikzcd}$$

Detailed description of SIDH

Public parameters:

- Prime $p = 2^{e_2}3^{e_3} - 1$
- Supersingular elliptic curve $E/\mathbb{F}_{p^2}$ of order $(p+1)^2$
- $\mathbb{Z}$-basis $\{P_2, Q_2\}$ of $E[2^{e_2}]$ and $\{P_3, Q_3\}$ of $E[3^{e_3}]$

Alice:

- Choose $\mathrm{sk}_2 \in \mathbb{Z}$ and compute $S_2 = P_2 + \mathrm{sk}_2 Q_2$ of order 2^{e_2}
- Compute $\phi_2 \colon E \to E/\langle S_2 \rangle$
- Send $E/\langle S_2 \rangle, \phi_2(P_3), \phi_2(Q_3)$ to Bob

Bob:

- Same as Alice, swapping 2 with 3

The shared secret is derived from

$$\begin{aligned} E/\langle S_2, S_3 \rangle &= (E/\langle S_2 \rangle)/\langle \phi_2(P_3) + \mathrm{sk}_3 \phi_2(Q_3) \rangle \\ &= (E/\langle S_3 \rangle)/\langle \phi_3(P_2) + \mathrm{sk}_2 \phi_3(Q_2) \rangle \end{aligned}$$

SIKE parameter sets

`SIKEp503:`

- $p = 2^{250}3^{159} - 1$ (note, the value of this prime is listed incorrectly in the spec)
- $P_2 = 3^{159} \cdot E(i+4),\ Q_2 = 3^{159} \cdot E(14)$
- $P_3 = 2^{250} \cdot E(i+7),\ Q_3 = 2^{250} \cdot E(6)$

`SIKEp751:`

- $p = 2^{372}3^{239} - 1$
- $P_2 = 3^{239} \cdot E(i+5),\ Q_2 = 3^{239} \cdot E(11)$
- $P_3 = 2^{372} \cdot E(i+1),\ Q_3 = 2^{372} \cdot E(6)$

`SIKEp964:`

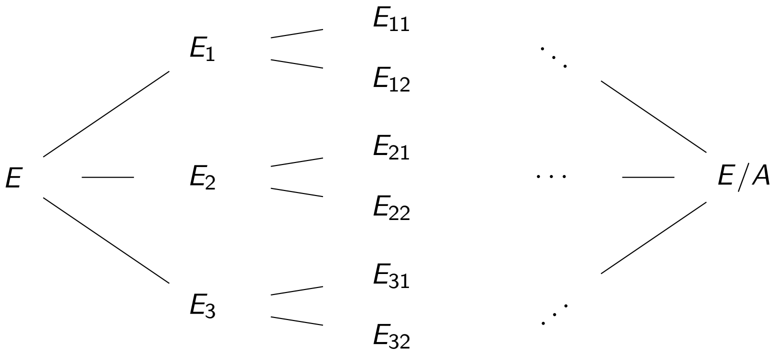
- $p = 2^{486}3^{301} - 1$
- $P_2 = 3^{301} \cdot E(i+23),\ Q_2 = 3^{301} \cdot E(11)$
- $P_3 = 2^{486} \cdot E(i+1),\ Q_3 = 2^{486} \cdot E(5)$

N.b.: $i = \sqrt{-1} \in \mathbb{F}_{p^2}$, $E : y^2 = x^3 + x$ and $E(x) = (x, \sqrt{x^3 + x})$.

Attack complexity

Hardness problem: Given E and E/A with a guarantee of the existence of $\phi\colon E \to E/A$, find A.

Fastest known (passive) attack is a generic collision search or claw search on a space of size $\deg(\phi)$:



Security

In principle, a non-generic attack against SIKE could conceivably exist; however, none is currently known. For **generic** attacks:

parameter set	security	NIST category
`SIKEp503`	`SHA256`	2
`SIKEp751`	`SHA384`	4
`SIKEp964`	`AES256/SHA512`	5

Recent developments pertaining to SIDH/SIKE security:

- Petit (Asiacrypt 2017): non-generic attacks against "unbalanced" versions of SIDH (**not used** in SIKE)
- Petit and Lauter, ePrint 2017/962: reductions from the isogeny problem to finding supersingular endomorphism rings
- Urbanik and Jao, AsiaPKC 2018: random self-reducibility
- Adj et al., ePrint:2018/313: proposes smaller parameters for 128-bit security, based on more detailed analysis of attacks

Implementation

0.069188618	s	KEX Total	[FrodoKEM-640]
0.075546943	s	KEX Total	[NTS-KEM(13, 80)]
0.114103121	s	KEX Total	[Ramstake RS 756839]
0.117327944	s	KEX Total	[ODD_MANHATTAN]
0.127024638	s	KEX Total	[RLCEKEM128B]
0.136131757	s	KEX Total	[DME-KEM (N=2, M=3, E=48, S=3)]
0.148760336	s	KEX Total	[NTS-KEM(13, 136)]
0.152088446	s	KEX Total	[FrodoKEM-976]
0.190694193	s	KEX Total	[SIKEp503]
0.646993100	s	KEX Total	[SIKEp751]
0.683500220	s	KEX Total	[CFPKM-128]
1.009693669	s	KEX Total	[Classic McEliece 8192128$]
1.214073736	s	KEX Total	[BIG_QUAKE_1]
1.679732008	s	KEX Total	[Classic McEliece 6960119]
2.033252376	s	KEX Total	[CFPKM-182]
2.334988284	s	KEX Total	[Post-Quantum RSA Enc - pqrsa15]
4.365430313	s	KEX Total	[BIG_QUAKE_3]
7.288352877	s	KEX Total	[DAGS_3]
8.105539551	s	KEX Total	[BIG_QUAKE]
52.913978368	s	KEX Total	[DAGS_5]

(credit: pqbench by Markku-Juhani O. Saarinen)

Key sizes:

- SIKEp503 — 378 bytes
- SIKEp751 — 564 bytes
- SIKEp964 — 726 bytes

- Performance with platform-specific Intel64 assembly optimizations (AVX2) is $\sim$ 9x faster
- Key compression (Zanon et al., PQCrypto 2018):
  - $\sim$ 40% smaller keys
  - $\sim$ 2x slower performance
  - Not included in SIKE specification, for the sake of simplicity

Summary

SIKE advantages:

- Very small key sizes
- No possibility for decryption error
- No complicated error distributions, rejection sampling, etc.
- Simple, conservative security analysis when assuming only generic attacks

SIKE disadvantages:

- Relatively slow
- Future analysis may uncover non-generic attacks against SIKE (though none are known so far)